Reg. No. : ....

Name : ....

K21U 1535

V Semester B.Sc. Degree (CBCSS – Sup./Imp.) Examination, November 2021
(2015 – '18 Admns)
CORE COURSE IN MATHEMATICS
5B08 MAT : Vector Calculus

Time : 3 Hours Max. Marks : 48

SECTION – A

All the **four** questions are **compulsory**. **Each** carries **1** mark.

1. Find the gradient of the function $f(x, y) = e^{\frac{y}{x}}$ at (1, 1).
2. Find the divergence of $F(x, y) = 12x^2yi - 3(x^3 - y^2)\, j$ at (1, 1).
3. Define the line integral of a function f(x, y, z) over a smooth curve C.
4. Find a vector equation of the curve $y = 9 - x^2$, $y \geq 0$, in the plane. **(4×1=4)**

SECTION – B

Answer **any 8** questions from questions **5** to **14**. **Each** carries **2** marks.

5. Write the vector equation of a line in the plane passing through (1, 1) and making an angle $\frac{\pi}{6}$ with the positive X axis.
6. Find the distance travelled by a particle when it moves along the curve $r(t) = 3t^2i - \sqrt{2}t^2j + 5t^2k$, $0 \leq t \leq 3$.
7. If $\omega = x^2 + y^2 + z^2$ and $z^3 + xy + yz + y^3 = 0$, find $\frac{\partial \omega}{\partial x}$ at (x, y, z) = (2, 1, 0) treating x and y as independent variables.
8. Find the equation to the tangent plane to the surface $3xy^2 = 4yz + 2$ at (2, 1, 1).
9. If F(x, y) = C, show that $\frac{dy}{dx} = -\frac{\left(\frac{\partial F}{\partial x}\right)}{\left(\frac{\partial F}{\partial y}\right)}$.

P.T.O.

10. Find the linear approximation $L(x, y)$ of the function $f(x, y) = (2x - y)^2$ at (1, 1).

11. Evaluate $\int_C (x+y)\, ds$ where C is given by $r(t) = ti + (1 - 3t)\, j$ from (0, 1) to (2, –5).

12. If the force $F = 3xi + 4yj + 3k$ is acting on a particle moving it along the curve C given by $r(t) = ti + (1 - t)j + tk$ from (0, 1, 0) to (1, 0, 1), find the work done by the force.

13. State the Gauss Divergence theorem.

14. Integrate $F(x, y, z) = x^2$ over the surface of the cone $z = \sqrt{x^2 + y^2}$, $0 \le z \le 1$. **(8×2=16)**

SECTION – C

Answer **any 4** questions from questions **15** to **20**. **Each** carries **4** marks.

15. Show that $\frac{d}{dt}(U \times V) = \frac{d}{dt}(U) \times V + U \times \frac{d}{dt}(V)$, where U, V are functions of t into $\mathbb{R}^2$.

16. Find the binormal vector to the curve $r(t) = \cos t\, i + \sin t\, j - k$ at $t = \frac{\pi}{4}$.

17. Suppose $D_U f(1, 2) = 10$ and $D_V f(1, 2) = 6$ where $U = \frac{3}{5}i - \frac{4}{5}j$ and $V = \frac{4}{5}i + \frac{3}{5}j$. Find $\frac{\partial f}{\partial x}(1, 2)$ and $\frac{\partial f}{\partial y}(1, 2)$.

18. Use the Taylor's formula for $f(x, y) = \cos(y + x)$ at the origin to find the quadratic approximation of f. Hence find approximate value of f(0.1, 0.2).

19. Find the scalar potential function of

$F(x, y, z) = (y^2 \cos x + z^3)i + (2y \sin x - 4)\, j + 3xz^2 k$.

20. Find the area of the surface cut from the bottom of the paraboloid $x^2 + y^2 - z = 0$ by the plane $z = 4$. **(4×4=16)**

SECTION – D

Answer **any 2** questions from questions **21** to **24**. **Each** question carries **6** marks.

21. If the equation of a curve is given by $r(t) = a \cos t\, i + a \sin t\, j + bt\, k$, find the unit tangent vector, unit normal vector and the curvature of the curve.

22. Using Lagrange's multiplier method, find the possible extreme points of $U(x, y) = x^2 + y^2$ subject to the constraint $x^2 + y^2 + 2x - 2y + 1 = 0$.

23. Verify Green's theorem for $\int_C (xy + y^2)\, dx + x^2 dy$ where C is the curve enclosing the region bounded by the parabola $y = x^2$ and the line $y = x$.

24. Verify Stoke's theorem for the vector field $F(x, y, z) = (2x - y)i - yz^2 j - y^2 zk$, taking the surface to be the upper half of the sphere $x^2 + y^2 + z^2 = 1$ and the curve to be its boundary on the XY-plane.

(2×6=12)
