K21U 4550

Reg. No. : .....

Name : .....

V Semester B.Sc. Degree CBCSS (OBE) Regular Examination, November 2021
(2019 Admn. Only)
CORE COURSE IN MATHEMATICS
5B05 MAT : Set Theory, Theory of Equations and Complex Numbers

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any four** questions from this Part. **Each** question carries **1** mark.

1. State the Uniqueness theorem.
2. Sum of the roots of the equation $x^3 - x - 1 = 0$ is _____.
3. If $1 + i$ is a root of a quadratic equation, then the other root will be _____.
4. What is a reciprocal equation ?
5. If the discriminant Δ of a cubic equation is negative, then it has _____.

PART – B

Answer **any eight** questions from this Part. **Each** question carries **2** marks.

6. If S is a finite set and $T \subseteq S$, then prove that T is finite.
7. Transform $x^3 - 6x^2 + 5x + 12 = 0$ into an equation which lacks the second term.
8. If α, β, γ are the roots of the equation $2x^3 + 3x^2 - x - 1 = 0$, then find the equation whose roots are $\alpha - 1, \beta - 1, \gamma - 1$.
9. State De Gua's rule.
10. Find an upper limit of the positive roots of the equation $x^3 - 10x^2 - 11x - 100 = 0$.
11. Write necessary and sufficient condition that the equation $ax^3 + 3bx^2 + 3cx + d = 0$ has two equal roots.
12. Discuss the character of the roots of the equation $x^3 + 29x - 97 = 0$ without finding them.
13. Explain the first and second kind reciprocal equations.
14. Express the complex number $2 + 2\sqrt{3}i$ in polar form.
15. Find $\text{Arg}(-1 - i)$.
16. State general form of De Movire's theorem.

P.T.O.

PART – C

Answer **any four** questions from this Part. **Each** question carries **4** marks.

17. State and prove Cantor's theorem.
18. Use Descartes rule of signs to show that $x^7 - 3x^4 + 2x^3 - 1 = 0$ has at least four imaginary roots.
19. If $a + b + c = 0$, then show that $a^5 + b^5 + c^5 = 5abc\,(ab + bc + ca)$.
20. Solve $6x^5 + 11x^4 - 33x^3 - 33x^2 + 11x + 6 = 0$.
21. Solve $y^3 - 7y^2 + 36 = 0$, where the difference between two of the roots is five.
22. For any two complex numbers a and b, prove that
$\left|a+\sqrt{a^2-b^2}\right|+\left|a-\sqrt{a^2-b^2}\right| = |a+b|+|a-b|$.
23. If $z = 1 + i$, then find $(1 + i)^{101}$.

PART – D

Answer **any two** questions from this Part. **Each** question carries **6** marks.

24. Prove that the set of all rational numbers is denumerable.
25. Find the rational roots of the equation $x^3 - 5x^2 - 18x + 72 = 0$.
26. Explain the Cardan's solution for general cubic equation.
27. Find all the fourth roots of unity and locate them graphically.
