K22U 0415

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS – OBE – Regular) Examination, April 2022
(2019 Admission)
CORE COURSE IN MATHEMATICS
6B12MAT : Numerical Methods, Fourier Series and Partial Differential Equations

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any four** questions. **Each** question carries **one** mark.

1. Write the Lagrange interpolation formula.
2. Solve the equation $y = x + y^2$ subject to the condition $y = 1$ when $x = 0$ using Picard's method.
3. Define a periodic function and find the period of $\sin \pi x$.
4. Verify that $u(x, t) = v(x + ct) + w(x - ct)$ with any twice differentiable function satisfy wave equation.
5. Define Fourier Transform of a non-periodic function f(x). **(4×1=4)**

PART – B

Answer **any eight** questions. **Each** question carries **2** marks.

6. Find $\log_{10} 301$ using Lagrange interpolation formula if certain corresponding values of x and $\log_{10} x$ are (300, 2.4771), (304, 2.4829), (305, 2.4843), (307, 2.4871).
7. If $f(x) = 2x^2 + 3x$, compute the entry $y = f(x)$ for x = 1, 3, 5, 7, 9 and prepare the forward difference table.
8. State the backward interpolation formula.

P.T.O.

9. Given the differential equation $y'' - xy' - y = 0$ with the conditions $y(0) = 1$ and $y'(0) = 0$, use Taylor's series method to determine the value of $y(0.1)$.

10. Using Euler's method, find $y(0.04)$, if $y' = -y$ with $y(0) = 1$.

11. Describe briefly the Picard's method of successive approximations.

12. Explain the Taylor's series method for solving differential equations with initial conditions.

13. State Euler formulas for Fourier coefficients.

14. Define even and odd functions. Prove that product of two odd functions is an even function.

15. Solve the PDE $u_{xx} + 16\pi^2 u = 0$.

16. Find the temperature $u(x, t)$ in a laterally insulated copper bar 80 cm long if the initial temperature is $100 \sin(\pi x/80)$°C and the ends are kept at 0°C. For copper, density is 8.92 g/cm^3, specific heat is 0.092 cal/(g°C) and thermal conductivity is 0.95 cal/(cm sec. °C).

(8×2=16)

PART – C

Answer **any four** questions. **Each** question carries **4** marks.

17. Using Newton's divided differences interpolation find f(x) as a polynomial if

x	–1	0	3	6	7
f(x)	3	–6	39	822	1611

18. Explain central difference table.

19. Using Newton's forward formula to compute the pressure of the steam at temperature 142° from the following steam table :

Temperature	140	150	160	170	180
Pressure	3.685	4.854	6.302	8.076	10.225

20. Explain different Runge-Kutta methods.

21. Determine the value of y when $x = 0.1$, given that $y(0) = 1$ and $y' = x^2 + y$ using Euler's method.

22. Find the Fourier series of the periodic function $f(x) = x^2, \ -\frac{1}{2} < x < \frac{1}{2}$.

23. Find the displacement of a string stretched between two fixed points at a distance 2c apart when the string is initially at rest in equilibrium position and points of the string are given initial velocities v where $v = \begin{cases} \frac{x}{c}, & 0 < x < c \\ \frac{2c - x}{c}, & c < x < 2c \end{cases}$ x being the distance measured from one end. **(4×4=16)**

PART – D

Answer **any two** questions. **Each** question carries **6** marks.

24. Estimate the population in 1895 and 1925 using Newton's interpolation formula from the following table :

Year : x	1891	1901	1911	1921	1931
Population : y	46	66	81	93	101

25. Given $\frac{dy}{dx} = y - x$ where $y(0) = 2$. Find $y(0.1)$ and $y(0.2)$ correct to four decimal places using Runge-Kutta fourth-order method.

26. a) Find the Fourier series expansion of the periodic function $f(x) = e^x, -\pi < x < \pi$ of period 2π.

b) Show that $\int_0^\infty \frac{1 - \cos \pi w}{w} \sin xw dw = \begin{cases} \frac{1}{2}\pi & \text{if } 0 << x < \pi \\ 0 & \text{if } x >> \pi \end{cases}$.

27. Prove that $u_{xx} + u_{yy} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}$ in polar co-ordinates. **(2×6=12)**
