



Reg. No. : .....

Name : .....



K21U 1125

IV Semester B.Sc. Degree CBCSS (OBE) Regular Examination, April 2021  
(2019 Admission Only)

CORE COURSE IN MATHEMATICS

4B04 MAT : Number Theory and Applications of Integrals

Time : 3 Hours

Max. Marks : 48

PART - A  
(Short Answer Type)

Answer any four out of five questions. Each question carries 1 mark.

1. State the Division Algorithm.
2. Find the lcm (31, 25).
3. Check whether the Diophantine equation  $6x + 51y = 22$  is solvable.
4. What is a pseudoprime ? Give an example.
5. State the Euler's theorem. (4x1=4)

PART- B  
(Short Essay Type)

Answer any eight out of eleven questions. Each question carries 2 marks.

6. If  $a|bc$ , with  $\gcd(a, b) = 1$ , show that  $a|c$ .
7. Find the gcd (12378, 3054).
8. For any choice of positive integers  $a$  and  $b$ , show that  $\text{lcm}(a, b) = ab$  if and only if  $\gcd(a, b) = 1$ .
9. Give an example to show that  $a^2 \equiv b^2 \pmod{n}$  need not imply that  $a \equiv b \pmod{n}$ .
10. If  $p$  is a prime, then show that  $a^p \equiv a \pmod{p}$  for any integer  $a$ .

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11. Evaluate  $\int_0^{\pi/4} \tan x \sec^2 x \, dx$ .
12. Find the length of the curve  $y = x^{3/2}$  from  $x = 0$  to  $x = 4$ .
13. Find the area of the region in the  $xy$ -plane enclosed by the cardioid  $r = 2(1 + \cos \theta)$ .
14. The region between the curve  $y = \sqrt{x}$ ,  $0 \leq x \leq 4$ , and the  $x$ -axis is revolved about the  $x$ -axis to generate a solid. Find its volume.
15. Find the volume of the solid generated by revolving the region between the  $y$ -axis and the curve  $x = \frac{2}{y}$ ,  $1 \leq y \leq 4$ , about the  $y$ -axis.
16. The circle  $x^2 + y^2 = a^2$  is rotated about the  $x$ -axis to generate a sphere. Find its volume. (8x2=16)

**PART - C**  
**(Essay Type)**

Answer **any four** out of seven questions. **Each** question carries **4** marks.

17. Determine all solutions in the integers of the Diophantine equation  $5x + 22y = 18$ .
18. For any positive integers  $a$  and  $b$ , show that  $a \equiv b \pmod{n}$  if and only if  $a$  and  $b$  leave the same remainder when divided by  $n$ .
19. If  $p$  is a prime, prove that for any integer  $a$ ,  
 $p \mid a^p + a(p-1)!$  and  
 $p \mid a^p(p-1)! + a$ .
20. Find the area of the region enclosed by the parabola  $y = 2 - x^2$  and the line  $y = -x$ .
21. Find the length of the graph of  $f(x) = \frac{x^2}{12} + \frac{1}{x}$ ,  $1 \leq x \leq 4$ .
22. The region bounded by the curve  $y = x^2 + 1$  and the line  $y = -x + 3$  is revolved about the  $x$ -axis to generate a solid. Find the volume of the solid.
23. Find the area of the surface generated by revolving the curve  $y = 2\sqrt{x}$ ,  $1 \leq x \leq 2$ , about the  $x$ -axis. (4x4=16)



PART - D  
(Long Essay Type)

Answer **any two** out of four questions. **Each** question carries **6** marks.

24. State and prove the Fundamental Theorem of Arithmetic.
  25. Prove that the quadratic congruence  $x^2 + 1 = 0 \pmod{p}$ , where  $p$  is an odd prime, has a solution if and only if  $p = 1 \pmod{4}$ .
  26. Find the area of the region in the first quadrant that is bounded above by  $y = \sqrt{x}$  and below by the  $x$ -axis and the line  $y = x - 2$ .
  27. Find the volume of the solid generated by revolving the region between the parabola  $x = y^2 + 1$  about the line  $x = 3$ . (2×6=12)
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