K23U 0230

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (C.B.C.S.S. – Supplementary) Examination, April 2023

(2017 to 2018 Admissions)

CORE COURSE IN MATHEMATICS

6B13MAT : Mathematical Analysis and Topology

Time : 3 Hours | Max. Marks : 48

SECTION – A

Answer **all** the questions, **each** question carries **1** mark.

1. If $P = \{a = x_0, x_1, x_2, \dots x_n = b\}$ is a partition of [a, b], then the Riemann lower sum of a function $f : [a, b] \to R$, is _________
2. Give an example of a sequence of continuous functions such that the limit function is not continuous.
3. A subset A of a topological space X is said to be dense if __________
4. Define the boundary point of a set A in a metric space X.

SECTION – B

Answer **any eight** questions, **each** question carries **2** marks.

5. If g(x) = x on [0, 1] and $P_n = \left\{0, \frac{1}{n}, \frac{2}{n}, \dots \frac{n-1}{n}, 1\right\}$ then find $\lim\limits_{n\to\infty} (U(P_n, g) - L(P_n, g))$.
6. If f is continuous on [a, b], a < b, show that there exist $c \in [a, b]$ such that we have $\int_a^b f = f(c)(b - a)$.
7. Give an example for a bounded non-integrable function on [0, 1].
8. Define pointwise convergence and uniform convergence of a sequence of functions.

P.T.O.

9. If f_n is continuous on $D \subseteq R$ and if $\sum f_n$ converges to f uniformly on D, prove that f is continuous on D.
10. Determine the radius of convergence of the power series $\sum \frac{n^n}{n!} x^n$.
11. Let X be a non-empty set and define d by
$$d(x, y) = \begin{cases} 0 & \text{if } x = y \\ 1 & \text{if } x \neq y \end{cases}$$
Show that d is a metric on X.
12. Prove that in a metric space X, each open sphere is an open set.
13. Prove that $\overline{A}$ equals the intersection of all closed supersets of A.
14. If T_1 and T_2 are 2 topologies on a non-empty set X, show that $T_1 \cap T_2$, is also a topology on X.

SECTION – C

Answer **any four** questions, **each** question carries **4** marks.

15. Show that if $f : [a, b] \to R$ is continuous on [a, b], then f is integrable on [a, b].
16. State and prove Darboux's theorem.
17. State and prove the Cauchy Criterion for Uniform Convergence.
18. Prove that every non-empty open set on the real line is the union of a countable disjoint class of open intervals.
19. Show that in a metric space X,
    a) any intersection of closed sets in X is closed.
    b) any finite union of closed sets in X is closed.
20. Show that a subset of a topological space is closed if and only if it contains its boundary.

SECTION – D

Answer **any two** questions, **each** question carries **6** marks.

21. If $f \in R[a, b]$ and if f is continuous at a point $c \in [a, b]$, prove that the indefinite integral $F(x) = \int_a^x f$ for $x \in [a, b]$ is differentiable at c and $F'(c) = f(c)$.

22. Prove that a sequence (f_n) of bounded functions on $A \subseteq R$ converges uniformly on A to f if and only if $\|f_n - f\|_A \to 0$.

23. State and prove Cantor's Intersection Theorem.

24. a) Let X and Y be topological spaces and f a mapping of X into Y. When do you say that f is :

    i) continuous

    ii) open

    iii) a homeomorphism ?

    b) Let X be a topological space, Y be a metric space, and A a subspace of X. If f is continuous mapping of A into Y, show that f can be extended in atmost one way to a continuous mapping of $\overline{A}$ into Y.