



Reg. No. : .....

Name : .....



K21U 1537

V Semester B.Sc. Degree (CBCSS – Sup./Imp.)  
Examination, November 2021  
(2017-'18 Admns.)  
CORE COURSE IN MATHEMATICS  
5B09 MAT : Graph Theory

Time : 3 Hours

Max. Marks : 48

PART – A

Answer all 4 questions :

(1×4=4)

1. Draw a graph on 4 vertices having a cut edge. Mark the cut edges.
2. Sketch 2 non isomorphic trees on 4 vertices.
3. Plot a tournament on 4 vertices.
4. Sketch a symmetric digraph on 4 vertices.

PART – B

Answer any 8 questions :

(2×8=16)

5. If  $G$  is simple and  $\delta \geq \frac{n-1}{2}$ , then prove that  $G$  is connected.
6. By considering two graphs  $G_1$  and  $G_2$  on 3 vertices, draw  $G_1 \cup G_2$  and  $G_1 \cap G_2$ .
7. If an edge  $e = xy$  of a connected graph  $G$  belongs to no cycle of  $G$ , prove that  $e$  is a cut edge of  $G$ .
8. Find the cut edges and the cut vertices of the graph given below.



9. Define the terms vertex connectivity and edge connectivity of a graph.

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10. By drawing a connected graph on 5 vertices, identify a spanning tree in it.
11. Justify the claim : A subset  $S$  of  $V$  is independent if and only if  $V - S$  is a covering of  $G$ .
12. Is the Wheel graph  $W_n$  Hamiltonian ? Justify your claim.
13. Define a strict digraph. Give an example of a strict digraph on 4 vertices.
14. Give an example of a strong digraph. Justify your answer.

PART - C

Answer any 4 questions :

(4×4=16)

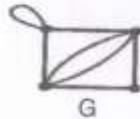
15. Show that if  $G$  is a self complementary graph of order  $n$ , then  $n \equiv 0, 1 \pmod{4}$ .
16. In any graph  $G$ , prove that number of vertices of odd degree is even.
17. Prove that any connected graph contains a spanning tree.
18. Show that a connected graph  $G$  with at least two vertices contains at least two vertices that are not cut vertices.
19. For a graph  $G$ , prove that  $\alpha' + \beta' = n$ .
20. Graphically explain the Königsberg bridge problem. Explain why the solution is not possible.

PART - D

Answer any 2 questions :

(6×2=12)

21. Show that a graph  $G$  is bipartite if and only if it contains no odd cycle.
22. Find  $\tau(G)$  of the graph.



23. Prove that a graph  $G$  is Eulerian if and only if each edge  $e$  of  $G$  belongs to odd number of cycles of  $G$ .
24. Prove that every tournament contains a directed Hamiltonian path.