K21U 2088

Reg. No. : ....

Name : ....

III Semester B.Sc. Degree (CBCSS – Sup.) Examination, November 2021
(2015 – 16 Admission)
CORE COURSE IN MATHEMATICS
3B03MAT – Elements of Mathematics – I

Time : 3 Hours Max. Marks : 48

SECTION – A

All the **first 4** questions are **compulsory**. They carry **1** mark **each**.

1. Write the statement in words '$\forall y \exists x(x + y \neq 0)$' where x, y consists of all integers.
2. State Fundamental theorem of Arithmetic.
3. If α, β, γ be the roots of the equation $2x^3 + x^2 - 22x - 24 = 0$ then $\alpha + \beta + \gamma =$.
4. Give an example of a polynomial having $x = 2$ is a root with multiplicity 3.

SECTION – B

Answer **any 8** questions from among the **5** to **14**. These questions carry **2** marks **each**.

5. Using De Morgan's law find the negation of the statement *No person has green eyes.*
6. Show that if C is an infinite set and B is a finite set, then C/B is an infinite set.
7. Solve $x^3 - 12x^2 + 39x - 28 = 0$ whose roots are in AP.
8. If α, β, γ be the roots of $x^3 + px^2 + qx + r = 0$, then prove that $(\alpha + \beta)(\beta + \gamma)(\gamma + \alpha) = r - pq$.
9. Define reciprocal equation, give one example.
10. Find sum $\sin x + \sin(2x) + \sin(3x) + ... + \sin(nx)$.
11. Find the Sturm's function $f_2(x)$ of $x^4 + 4x^3 - 4x - 13 = 0$.

P.T.O.

12. If $a \equiv b \pmod{n}$ and $c > 0$ then show that $ca \equiv cb (\text{mod } cn)$.

13. Find the remainders when 2^{50} and 41^{65} are divisible by 7.

14. If gcd (a, b) = d then show that $\gcd\left(\frac{a}{d}, \frac{b}{d}\right) = 1$.

SECTION – C

Answer **any 4** questions from among the **15** to **20**. These questions carry **4** marks **each**.

15. Show that $p \to q \equiv -p \vee q$.

16. If α, β, γ are the roots of the equation $x^3 + qx + r = 0$. Find the equation whose roots are $\alpha(\beta+\gamma), \beta(\gamma+\alpha), \gamma(\alpha+\beta)$.

17. Show that, if the roots of the equation $ax^3 + bx^2 + cx + d = 0$, are in GP then $c^3a + b^3d$.

18. Solve $x^3 - x^2 - 8x + 12 = 0$ given that it has a double root.

19. Using the Siêve of Eratosthenes find all primes not exceeding 60.

20. Show that any integer of the form $6k + 5$ is also of the form $3j + 2$ but not converse.

SECTION – D

Answer **any 2** questions from among the **21** to **24**. These questions carry **6** marks **each**.

21. i) State and prove Cantor's theorem.

    ii) Exhibit a bijection between $\mathbb{N}$ and a proper subset of itself.

22. Obtain a cubic equation whose roots are α, β, γ given that $\alpha + \beta + \gamma = 3$, $\alpha^2 + \beta^2 + \gamma^2 = 5$, $\alpha^3 + \beta^3 + \gamma^3 = 7$.

23. Solve $x^4 + 2x^3 - 7x^2 - 8x + 12 = 0$.

24. Show that congruence is an equivalence relation.
