



K21U 1534

Reg. No. :

Name :

V Semester B.Sc. Degree (CBCSS – Sup./Imp.) Examination, November 2021
(2015-'18 Admns.)

CORE COURSE IN MATHEMATICS

5B07MAT : Differential Equations, Laplace Transform and Fourier Series

Time : 3 Hours

Max. Marks : 48

PART – A

Answer all 4 questions.

(1×4=4)

1. Find an integrating factor of the differential equation $xdy - ydx = 0$.
2. Evaluate the Wronskian of $y_1 = \cos t$, $y_2 = \sin t$.
3. Write the Laplace transform of $e^{at} \cos bt$.
4. Justify your answer the function $f(x) = x^2 \cos nx$ is even.

PART – B

Answer any 8 questions.

(2×8=16)

5. Solve the initial value problem $y' = -2xy$, $y(0) = 1$.
6. Find the value of b for which the following equation is exact :
 $(xy^2 + bx^3y)dx + (x + y) x^2dy = 0$.
7. Obtain the differential equation associated with the primitive $y = Ax^2 + Bx + C$.
8. Find a particular solution of $y'' - 4y' - 4y = 2e^{2x}$.
9. Find the general solution of $(9D^2 - 1)y = 0$, where D is the differential operator.
10. Write the Laplace transform of the function $f(t) = \begin{cases} e^t, & 0 < t < 1 \\ 0, & 1 < t < \infty \end{cases}$.

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11. Evaluate $L\left(\frac{1-e^t}{t}\right)$.
12. Find the inverse Laplace transform of the function $\frac{1}{s^2 - 4s + 5}$.
13. Show that the product of two odd functions is even.
14. Sketch the graph of the function $f(x) = 1 - x^2$ if $-1 \leq x \leq 1$ and $f(x+2) = f(x)$.

PART - C

Answer any 4 questions.

(4×4=16)

15. Solve the differential equation $y^2 y' - y^2 \tan x = \sin x \cos^2 x$.
16. Given that Y_1 and Y_2 are solutions of the non-homogeneous equation $y'' + p(t)y' + q(t)y = g(t)$. Prove that $Y_1 - Y_2$ is a solution of the corresponding homogeneous equation $y'' + p(t)y' + q(t)y = 0$.
17. By method of variation of parameters, solve the differential equation, $y'' + y = \tan x$.
18. Assuming the required conditions, prove that $L[f'(t)] = sL[f(t)] - f(0)$.
19. Find the Fourier sine series expansion of $f(x) = 2 - x$ when $0 < x < 2$ with period 4.
20. Find the Fourier cosine integral representation of the function $f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$.

PART - D

Answer any 2 questions.

(6×2=12)

21. Find the orthogonal trajectories of the families of curves $\frac{1}{2}x^2 + y^2 = c$.
22. Using the method of undetermined coefficients, solve the differential equation $y'' - y = 2t^2$.
23. State and prove convolution theorem for Laplace transform.
24. Find the Fourier series of the function $f(x) = x^2$ if $-\pi < x < \pi$ and $f(x+2\pi) = f(x)$.