K22U 0414

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS – OBE – Regular) Examination, April 2022
(2019 Admission)
CORE COURSE IN MATHEMATICS
6B11 MAT : Complex Analysis

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any four** questions. **Each** question carries **one** mark.

1. Find the real and imaginary parts of the function $f(z) = \frac{1}{z}$.
2. Evaluate $\int_0^{1+i} z^2 \, dz$.
3. State Morera's theorem.
4. Write the Laurent series for $z^2 e^{1/z}$.
5. Find residue of $f(z) = \frac{\sin z}{z^4}$.

PART – B

Answer **any eight** questions. **Each** question carries **two** marks.

6. Solve cos z = 5.
7. Find the Principal value of ln(i).
8. Evaluate $\int_C \operatorname{Re}(z) \, dz$, where $C : z(t) = t + 2it, (0 \le t \le 1)$.
9. Show that the fundamental region of e^z is $-\pi < y \le \pi$.
10. Find an upperbound for the absolute value of $\int_C z^2 \, dz$.

P.T.O.

11. State identity theorem for power series.

12. Define absolute convergence and conditional convergence.

13. Check the convergence of $\sum_{n=0}^{\infty} \frac{(100+75i)^n}{n!}$.

14. Show that sequence $\{z_n = x_n + iy_n\}$ converges to $c = a + ib$ if and only if $\{x_n\}$. converges to a and $\{y_n\}$ converges to b.

15. State Picard's theorem.

16. $\int_C \frac{z^3-6}{2z-i} dz$ where C is $|z| = \frac{3}{4}$.

PART – C

Answer **any four** questions. **Each** question carries **four** marks.

17. Verify $u = x^2 - y^2 - y$ is harmonic and find the harmonic conjugate of u.

18. Find $(1 + i)^{2-i}$.

19. State and prove Cauchy's inequality.

20. State and prove Liouville's Theorem.

21. Find radius of convergence of the following.

    a) $\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2} (z-3i)^n$.

    b) $\left[(-1)^n + \frac{1}{2^n}\right] z^n$.

22. Find residue at poles of the function $f(z) = \frac{9z+i}{z^3+z}$.

23. Classify isolated singularities. Give suitable examples too.

PART – D

Answer **any two** questions. **Each** question carries **six** marks.

24. a) State and prove necessary condition for differentiability.

    b) If f is an analytic function with $|f|$ constant, then show that f is constant.

25. a) State Cauchy's Integral Formula.

    b) $\int_C \frac{z^2+1}{z^2-1} dz$, where C is $|z-1|=1$.

    c) $\int_C \frac{\tan z}{z^2-1} dz$, where C is $\left|z-\frac{\pi}{2}\right|=\frac{1}{4}$.

26. a) Find Maclaurin's series for $f(z)=\frac{1}{(1+z)^2}$.

    b) Find Taylor's series for $f(z)=\frac{2z^2+9z+5}{z^3+z^2-8z-12}$.

27. a) State and prove Cauchy Residue Theorem.

    b) Evaluate $\int_C \left(\frac{ze^{\pi z}}{z^4-16}+ze^{\frac{\pi}{z}}\right)dz$, where C is the ellipse $9x^2+y^2=9$.
