K21U 1832

Reg. No. : ....

Name : ....

III Semester B.Sc. Degree CBCSS (OBE) Reg./Sup./Imp. Examination, November 2021
(2019 – 2020 Admission)
CORE COURSE IN MATHEMATICS
3B03 MAT : Analytic Geometry and Applications of Derivatives

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any four** questions. **Each** question carries **one** mark.

1. Find the eccentricity of the ellipse $2x^2 + y^2 = 2$.

2. Evaluate $\lim_{x \to 0} \frac{\sin 5x}{x}$.

3. Find the angle ϕ between the radius vector and the tangent at any point on the curve $r = a(1 - \cos\theta)$.

4. Write the formula for finding the radius of curvature for a polar curve $r = f(\theta)$.

5. Define asymptote of a curve.

PART – B

Answer **any eight** questions. **Each** question carries **two** marks.

6. Find the focus and directrix of the parabola $y^2 = -4x$.

7. Find the equation of ellipse with Foci : $\left(\pm\sqrt{2}, 0\right)$ Vertices : $(\pm 2, 0)$.

8. Find the critical points for the function $f(x) = 6x^2 - x^3$.

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9. Evaluate $\lim\limits_{x \to \pi/2} \frac{\sec x}{1+\tan x}$.

10. Find the absolute maximum and minimum values of $f(x) = 4 - x^2, -3 \leq x \leq 1$.

11. Determine the concavity of $y = 3 + \sin x$ on $[0, 2\pi]$.

12. Verify Rolle's Theorem for the function $y = e^x (\sin x - \cos x)$ in $(\pi/4, 5\pi/4)$.

13. Find the asymptotes of the curve $x^2y^2 - x^2y - xy^2 + x + y + 1 = 0$.

14. Find ρ at the origin for the curve $y^4 + x^3 + a(x^2 + y^2) - a^2y = 0$.

15. Find the polar subtangent of the cardioid $r = a(1 - \cos\theta)$.

16. Show that the parabolas $y^2 = 4ax$ and $2x^2 = ay$ intersect at an angle of $\tan^{-1}(3/5)$.

PART – C

Answer **any four** questions. **Each** question carries **four** marks.

17. Sketch the hyperbola $y^2 - x^2 = 4$ including asymptotes and foci.

18. Find a Cartesian equation for the hyperbola centered at the origin that has a focus at (3, 0) and the line $x = 1$ as the corresponding directrix.

19. A particle is moving along a horizontal coordinate line (positive to the right) with position function $s(t) = 2t^3 - 14t^2 + 22t - 5$, $t \geq 0$. Find the velocity and acceleration.

20. Prove that $\lim\limits_{x \to 0^+} (1 + x)^{1/x} = e$.

21. Find the equation of the tangent at any point (x, y) to the curve $x^{2/3} + y^{2/3} = a^{2/3}$. Show that the portion of the tangent intercepted between the axes is of constant length.

22. Find the angle of intersection of the curves : $r = \sin\theta + \cos\theta$, $r = 2\sin\theta$.

23. Find the asymptotes of $r = a\tan\theta$.

PART – D

Answer **any two** questions. **Each** question carries **6** marks.

24. Derive the polar equation of a conic with eccentricity e. Also find the directrix of the parabola $r = \frac{25}{10 + 10\cos\theta}$.

25. Evaluate the following :

a) $\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$.

b) Find the critical points of $f(x) = x^3 - 12x - 5$ and identify the intervals on which f is increasing and on which f is decreasing.

26. Define Evolute. Show that the evolute of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ is another equal cycloid.

27. Find the lengths of the tangent, normal, subtangent and subnormal for the cycloid : $x = a(t + \sin t)$, $y = a(1 - \cos t)$.

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