



Reg. No. :

Name :



K19U 2255

V Semester B.Sc. Degree (CBCSS- Reg./Sup./Imp.) Examination,
November - 2019

(2014 Admn. Onwards)

Core Course in Mathematics

5B 06 MAT: ABSTRACT ALGEBRA

Time : 3 Hours

Max. Marks : 48

SECTION - A

Answer **All** Questions, Each question carries **One** Mark. (4×1=4)

1. Is the usual addition a binary operation on the set of all prime numbers? Justify your answer.
2. Define orbits of a permutation σ of a set A .
3. Define normal subgroup of a group G . Given an example.
4. What is the characteristic of the ring of real numbers under usual addition and multiplication?

SECTION - B

Answer any **Eight** Questions, Each question carries **Two** Marks.
(8×2=16)

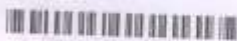
5. Prove that every cyclic group is abelian.
6. Show that a nonempty subset H of group G is a subgroup of G if and only if $ab^{-1} \in H$ for all $a, b \in H$.
7. Describe S_n , the symmetric group on n letters.
8. Find the index of $\langle 3 \rangle$ in the group Z_n .

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K19U 2255



(2)



9. Prove that the identity permutation in S_n is an even permutation for $n \geq 2$.
10. Determine the number of group homomorphisms from Z onto Z .
11. Find the characteristic of the ring $Z_n \times Z_m$.
12. Prove that every field is an integral domain.
13. Define a ring and give an example of a finite ring which is not an integral domain.
14. Find the remainder of 8^{100} when divided by 13.

SECTION - C

Answer any **Four** Questions, Each question carries **four** Marks. (4x4=16)

15. Show that every finite cyclic group of order n is isomorphic to $\langle Z_n, +_n \rangle$.
16. Show that the set of all permutations of any nonempty set A is group under permutation multiplication.
17. Prove that every group of prime order is cyclic.
18. Let φ be a homomorphism of a group G into G' . Then prove the following:
 - a) If $a \in G$, prove that $\varphi(a^{-1}) = (\varphi(a))^{-1}$
 - b) If H is a subgroup of G , then $\varphi[H]$ is a subgroup of G'
19. Prove that every finite integral domain is a field.
20. Prove that in the ring Z_n the divisors of 0 are precisely those nonzero elements that are not relatively prime to n .

SECTION - D

Answer any **Two** Questions, Each Question carries **Six** Marks. (2x6=12)

21. a) Define the greatest common divisor of two positive integers. Also find the quotient and remainder when 50 is divided by 8 according to division algorithm.
- b) Prove that subgroup of a cyclic group is cyclic.



(3)



K19U2255

22. a) State and prove Lagrange's Theorem.
b) Prove that the collection of all even permutations of $\{1, 2, 3, \dots, n\}$ forms a subgroup of order $\frac{n!}{2}$ of the symmetric group S_n ; $n \geq 2$.
23. a) State and prove the fundamental homomorphism theorem.
b) Show that a group homomorphism is one-one if and only if its kernel consists of only the identity element.
24. a) Show that the cancellation law holds in a ring iff it has no divisors of 0.
b) Show that the matrix $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ is a divisor of 0 in $M_2(\mathbb{Z})$.
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