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K17U 1696

Reg. No. :

V Semester B.Sc. Degree (CBCSS - Reg./Sup./Imp.)
Examination, November 2017
(2014 Admn. Onwards)
CORE COURSE IN MATHEMATICS
5B06 MAT: Abstract Algebra

Time: 3 Hours

Max. Marks: 48

SECTION-A

Answer all the questions. Each question carries one mark.

- Find the number of elements in the cyclic subgroup of Z₃₀ generated by 25.
- 2. What are the orbits of the identity permutation o of a set A?
- 3. How many homomorphisms are there of ≥ onto ≥ ?
- 4. What are the units in 2×2?

 $(4 \times 1 = 4)$

SECTION-B

Answer any 8 questions. Each question carries two marks.

- 5. Show that every permutation of a finite set is a product of disjoint cycles.
- Let H be a subgroup of a finite group G. Show that the order of H is a divisor of the order of G.
- Show that if σ is a cycle of odd length, then σ² is a cycle.
- Find all cosets of the subgroup (2) of Z₁₂.
- 9. Express the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 2 & 6 & 3 & 7 & 4 & 5 & 1 \end{pmatrix}$ as a product of disjoint cycles
 - and then as a product of transpositions.

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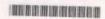
- 10. Determine the order of the element $5 + \langle 4 \rangle$ in $\mathbb{Z}_{12}/\langle 4 \rangle$.
- Show that in the ring Z_n, the divisors of 0 are precisely those nonzero elements that are not relatively prime to n.
- 13. Show that every finite integral domain is a field.
- If a ∈ Z and p is a prime not dividing a, show that p divides a^{p-1} 1. (8x2=16)

SECTION - C

Answer any 4 questions. Each question carries four marks.

- 15. Show that every subgroup of a cyclic group is cyclic.
- 16. State and prove Lagrange's theorem. Deduce that the order of an element of a finite group divides the order of the group.
- 17. Let G and G' be groups and let φ: G → G' be a one-to-one function such that φ(xy)=φ(x)φ(y) for all x, y ∈ G. Show that φ[G] is a subgroup of G' and φ provides an isomorphism of G with φ[G].
- Let φ: G → H be a group homomorphism. Show that φ[G] is abelian if and only if for all x, y ∈ G we have xyx⁻¹y⁻¹ ∈ Ker(φ).
- Describe all ring homomorphisms of Z into Z x Z.
- 20. Find all solutions of the congruence 15x = 27 (mod 18).

 $(4 \times 4 = 16)$





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Answer any 2 questions. Each question carries six marks.

- 21. If a is a generator of a finite cyclic group G of order n, show that the other generators of G are the elements of the form a^r, where r is relatively prime to n.
- $\mathcal{A}2$. List the elements of the symmetric group S_3 on 3 letters and form the multiplication table for S_3 . Find all subgroups of S_3 .
 - 23. State and prove the fundamental homomorphism theorem.
 - Show that the cancellation laws hold in a ring R if and only if R has no divisors of 0.
 - b) Show that $a^2 b^2 = (a + b) (a b)$ for all a and b in a ring R if and only if R is commutative.
 - c) Show that 1 and p − 1 are the only elements of the field Z_p that are their own multiplicative inverse.
 (2×6=12)