K23U 0229

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (C.B.C.S.S. – Supplementary) Examination, April 2023
(2017 to 2018 Admissions)
CORE COURSE IN MATHEMATICS
6B12MAT : Complex Analysis

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** the questions. **Each** question carries **1** mark.

1. Find $(1 + i)^{16}$.

2. Determine the principal value of the argument of $-5 - 5i$.

3. State Taylor's theorem.

4. Develop a Maclaurin series of the function $\frac{1}{1-z^4}$.

SECTION – B

Answer **any eight** questions. **Each** question carries **2** marks.

5. Write $\frac{z_1 - z_2}{z_1 + z_2}$ of the form $x + iy$, where $z_1 = 4 + 3i$ and $z_2 = 2 - 5i$.

6. If $z = x + iy$, show that $\sin z = \sin x \cosh y + i \cos x \sinh y$.

7. Find the principal value of i^i.

8. Evaluate $\int_{8+\pi i}^{8-3\pi i} e^{\frac{z}{2}} dz$.

9. Integrate $\frac{z^2}{z^4 - 1}$ counter clockwise around the circle $|z + 1| = 1$.

P.T.O.

10. Integrate $f(z) = \dfrac{z^3 + \sin z}{(z-i)^2}$ counter clockwise around the boundary of the square with vertices ± 2 and $\pm 2i$.

11. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \dfrac{n+5i}{(2n)!}(z-i)^n$.

12. Is the series $\sum_{n=1}^{\infty} \dfrac{(n!)^2}{(2n)!}$ convergent ? Justify your answer.

13. Determine the location and type of singularity of the function cot 2z.

14. Find $\text{Res}_{z=i} \dfrac{9z+i}{z(z^2+1)}$.

SECTION – C

Answer **any four** questions. **Each** question carries **4** marks.

15. Verify triangle inequality for $z_1 = 4 - 6i$, $z_2 = 2 + 3i$.

16. If f(z) is analytic in a simply connected domain D, then show that the integral of f(z) is independent of path in D.

17. Show that $\int_C \dfrac{dz}{(z-z_1)(z-z_2)} = 0$ for a simple closed path C enclosing z_1 and z_2.

18. State and prove root test for convergence of series.

19. Determine the location and order of the zero of $(z^4 - z^2 - 6)^3$.

20. Using Residue theorem, evaluate $\int_C \dfrac{z+1}{z^4 - 2z^3}\, dz$ where C is the circle $|z| = \dfrac{1}{2}$ (Counter clockwise).

SECTION – D

Answer **any two** questions. **Each** question carries **6** marks.

21. Find all solutions of :
    a) $e^z = 1$
    b) $\cos z = 3i$.

22. State and prove M-L inequality. Using this show that $\left|\int_C \frac{dz}{z^4}\right| \leq 4\sqrt{2}$ where C denote the line segment from z = i to z = 1.

23. a) Prove that a sequence $z_1, z_2, ..., z_n, ...$ of complex numbers $z_n = x_n + iy_n$ (where n = 1, 2, ...) converges to c = a + ib if and only if the sequence of real parts $x_1, x_2, ...$ converges to a and the sequence of imaginary parts $y_1, y_2, ...$ converges to b.

    b) Is the sequence $z_1, z_2, ..., z_n, ...$ where $z_n = \frac{n\pi i}{n+i}$ converges ? Justify.

24. Find all Taylor and Laurent series of $f(z) = \frac{-2z+3}{z^2 - 3z + 2}$ with center 0.