K23U 0228

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS – Supplementary) Examination, April 2023
(2017 to 2018 Admissions)
CORE COURSE IN MATHEMATICS
6B11MAT : Numerical Methods and Partial Differential Equations

Time : 3 Hours | Max. Marks : 48

SECTION – A

Answer **all** the questions. **Each** question carries **1** mark.

1. Give Newton's forward difference interpolation polynomial.
2. Write down the two dimensional Laplace equation.
3. Find the order of the partial differential equation $\frac{\partial^3 u}{\partial x^3} + \left(\frac{\partial u}{\partial x}\right)^2 = 0$.
4. Show that $u = e^{-t} \sin x$ is a solution of the differential equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$.

SECTION – B

Answer **any eight** questions. **Each** question carries **2** marks.

5. Show that the smallest positive root of $x^3 - 3x - 1 = 0$ lies in the interval (1, 2).
6. Using the method $f'(x_0) = \frac{1}{2h}[-3f_0 + 4f_1 - f_2]$, obtain an approximate value of $f'(-3)$ with h = 2, for the following data :

x	–3	–2.5	–2	–1	1
f(x)	–25	–14.125	–7	–1	–1

7. Evaluate $\int_0^2 \frac{dx}{x^2 + 2x + 10}$, using trapezoidal rule with n = 2.

P.T.O.

8. Construct the divided difference table for the following data :

x	–3	–2	–1
f(x)	–25	–7	–1

9. By performing two iterations of the bisection method, obtain the smallest positive root of the equation $x^3 - 5x + 1 = 0$.

10. Prove that $\Delta\left(\frac{1}{f_i}\right) = -\frac{\Delta f_i}{f_i f_{i+1}}$.

11. Explain the terms Quadratic rule and Error of approximation in numerical integration.

12. Solve the partial differential equation $y^2u_x - x^2u_y = 0$, by separating variables method.

13. Verify that $u = x^2 + y^2$, $f = 4$ satisfies the partial differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f$.

14. Find the solution of the initial value problem $y' = x + y$, $y(0) = 1$, by performing two iterations of Picard's method.

SECTION – C

Answer **any four** questions. **Each** question carries **4** marks.

15. Find the approximate value of y(1.2) for the IVP $y' = -2xy^2$, $y(1) = 2$, using Taylor's second order method.

16. Find u(x, t) of the string of length π, $c^2 = 1$, initial velocity zero and initial deflection $0.1x(\pi^2 - x^2)$.

17. Find the Lagrange interpolation polynomial that fits the following data values :

x	1	2	4
f(x)	1	7	61

18. Using Newton Raphson method, find the value of $18^{\frac{1}{3}}$ upto four decimal places taking suitable initial approximation.

19. Transform the equation $u_{xx} + 4u_{xy} + 4u_{yy} = 0$ into normal form and solve.

20. Evaluate $\sqrt{3}$ using the equation $x^2 - 3 = 0$ by applying the fixed point iteration method.

SECTION – D

Answer **any two** questions. **Each** question carries **6** marks.

21. Find the value of $\int_0^1 \frac{dx}{2+3x}$ using Simpson's rule with n = 2 and compare this value with the exact solution.

22. Find the solution of one dimensional heat equation by using Fourier series.

23. Solve the initial value problem, $y' = x(y - x)$, $y(1) = 2$ in the interval [1, 1.2] using the classical Runge-Kutta fourth order method with the step size h = 0.1.

24. The following table represents the function $f(x) = e^{-x}$.

x	–1	–0.5	0	0.5	1
f(x)	2	0.75	1	2.75	6

i) Using Gauss forward central difference formula, compute f(0.25).

ii) Using Gauss backward central difference formula, compute f(–0.25).