K23U 0227

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (C.B.C.S.S. – Supplementary) Examination, April 2023
(2017 to 2018 Admissions)
CORE COURSE IN MATHEMATICS
6B10MAT – Linear Algebra

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** the questions, **each** question carries **1** mark.

1. Define maximal linearly independent subset of a vector space.
2. Find the null space of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ by T(a, b, c) = (a, – b, 2c).
3. Define characteristic root and characteristic vector of a square matrix A.
4. Find the eigenvalues of the matrix, $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$.

SECTION – B

Answer **any eight** questions, **each** question carries **2** marks.

5. Define vector space and give an example.
6. Define basis of a vector space and write a basis of $M_{2\times2}(F)$.
7. Let V and W be vector spaces and let T : V → W be linear. Show that T is one-to-one if and only if N(T) = {0}.
8. Let α and β be standard ordered basis for $\mathbb{R}^2$ and $\mathbb{R}^3$ respectively. Define $T : \mathbb{R}^2 \to \mathbb{R}^3$ by T (a, b) = (a + 3b, 0, 2a – 4b). Find the matrix $[T]_\alpha^\beta$.

P.T.O.

9. State and prove Sylvester's law of nullity.

10. Show that the characteristic roots of an idempotent matrix are either zero or unity.

11. Find the product of the characteristic roots of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

12. If A is non-singular, prove that the eigenvalues of A^{-1} are the reciprocals of the eigenvalues of A.

13. Use Gauss elimination to solve the system

$2x + y + z = 8$

$3x + 2y + 3z = 18$

$x + 4y + 9z = 28$

14. Define eigenspace of a linear operator T.

SECTION – C

Answer **any four** questions, **each** question carries **4** marks.

15. Let $\mathcal{F}(\mathbb{R}, \mathbb{R})$ be the vector space of all functions from $\mathbb{R}$ to $\mathbb{R}$. Show that the set $C(\mathbb{R})$ of all real valued continuous functions defined on $\mathbb{R}$ is a subspace of $\mathcal{F}(\mathbb{R}, \mathbb{R})$.

16. If S is a nonempty subset of a vector space V, show that the set W consisting of all linear combinations of elements of S is a subspace of V and W is the smallest subspace containing S.

17. Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is linear and that T(0, 1) = (1, 4) and T(1, 1) = (2, 5). What is T(2, 3) ?

18. Find a basis of the solution space of the system of equations

$x + y - z + t = 0$

$x - y + 2z - t = 0$

$3x + y + t = 0$

19. If A and B are two square matrices, show that the matrices AB and BA have the same characteristic roots.

20. Use Gauss-Jordan method to solve the system

$4x_1 + 3x_2 - x_3 = 6$

$3x_1 + 5x_2 + 3x_3 = 4$

$x_1 + x_2 + x_3 = 1$

SECTION – D

Answer **any two** questions, **each** question carries **6** marks.

21. Let S be a linearly independent subset of a vector space V. Prove that there exists a maximal linearly independent subset of V that contains S.

22. State and prove Dimension theorem.

23. Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and find A^{-1}.

24. Show that $A = \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix}$ is diagonalizable.