K23U 2829

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (C.B.C.S.S. – Supplementary) Examination, November 2023
(2017 and 2018 Admissions)
CORE COURSE IN MATHEMATICS
5B09 MAT : Graph Theory

Time : 3 Hours Max. Marks : 48

PART – A
(Short Answer)

Answer **all** questions from this Part. **Each** question carries **1** mark. **(4×1=4)**

1. Define graph isomorphism.
2. Define normal product of two graphs and find $n(G_1 \circ G_2)$.
3. State Whitney's theorem on 2-connected graphs.
4. Give an example of a graph with n vertices and n – 1 edges that is not a tree.

PART – B
(Short Essay)

Answer **any eight** questions from this Part. **Each** question carries **2** marks. **(8×2=16)**

5. State and prove the first theorem of graph theory.
6. Let $(d_1, d_2, ..., d_n)$ be the degree sequence of a graph and r be any positive integer. Show that $\sum_{i=1}^{n} d_i^r$ is even.
7. Prove that the line graph of a simple graph G is a path if and only if G is a path.
8. Prove that a vertex v of a connected graph with at least three vertices is a cut vertex of G if and only if there exist vertices u and w of G, distinct from v, such that v is in every u – w path in G.
9. Disprove by a counter example : If k(G) = k, then (L(G)) = k.
10. Prove that a simple graph is a tree if and only if any two distinct vertices are connected by a unique path.
11. If $\delta(G) \geq 2$, then prove that G contains a cycle.

P.T.O.

12. Prove that a subset S of V is independent if and only if V – S is a covering of G.

13. For any graph G with $\delta > 0$, prove that $\alpha \leq \beta'$ and $\alpha' \leq \beta$.

14. Explain directed graph with an example.

PART – C
(Essay)

Answer **any four** questions from this Part. **Each** question carries **4** marks. **(4×4=16)**

15. If G is simple and $\delta \geq \frac{n-1}{2}$, then prove that G is connected. Give an example of a non-simple disconnected graph with $\delta \geq \frac{n-1}{2}$.

16. Prove that a connected graph G with at least two vertices contains at least two vertices that are not cut vertices.

17. Prove that for a simple connected graph G, L(G) is isomorphic to G if and only if G is a cycle.

18. For any graph G for which $\delta > 0$, prove that $\alpha' + \beta' = n$.

19. If G is Hamiltonian, then prove that for every nonempty proper subset S of V, $\omega(G - S) \leq |S|$.

20. Show that every tournament T is disconnected or can be made into one by the reorientation of just one arc of T.

PART – D
(Long Essay)

Answer **any two** questions from this Part. **Each** question carries **6** marks. **(2×6=12)**

21. a) Prove that a simple non-trivial graph G is connected if and only if for any partition of V into two non-empty subsets V_1 and V_2, there is an edge joining a vertex of V_1 to a vertex of V_2.

    b) Prove that in a connected graph G with at least three vertices, any two longest paths have a vertex in common.

22. For any loopless connected graph G, prove that $k(G) \leq \lambda(G) \leq \delta(G)$.

23. For a connected graph G, prove that G is Eulerian if and only if the degree of each vertex of G is an even positive integer.

24. Prove that every tournament contains a directed Hamiltonian path.