Reg. No. : $\qquad$
Name: $\qquad$
V Semester B.Sc. Degree (CBCSS - O.B.E. - Regular/Supplementary/ Improvement) Examination, November 2023 (2019-2021 Admissions) CORE COURSE IN MATHEMATICS
5B08 MAT : Differential Equations and Laplace Transforms
Time : 3 Hours
Max. Marks : 48

## PART - A <br> (Short Answer)

Answer any four questions from this Part. Each question carries 1 mark. ( $4 \times 1=4$ )

1. Solve the differential equation $y^{\prime}=1+y^{2}$.
2. Check whether the equation $-y d x+x d y=0$ is exact.
3. Give an example of a non-homogeneous differential equation.
4. Solve $y^{\prime \prime}-\mathrm{y}=0$.
5. State the linearity property of the Laplace transform.
PART - B
(Short Essay)
Answer any eight questions from this Part. Each question carries 2 marks. (8×2=16)
6. Find the order and degree of the differential equation $\frac{d^{2} y}{d x^{2}}+\left[1+\left(\frac{d y}{d x}\right)^{2}\right]^{\frac{3}{2}}=0$.
7. Prove that $e^{x}$ is an integrating factor of siny $d x+\cos y d y=0$ and solve it.
8. Find the orthogonal trajectories of the curve $y=c^{-x}$.
9. State the existence theorem of first order differential equations.
10. Solve the initial value problem $y^{\prime \prime}-y^{\prime}-2 y=0, y(0)=-4, y^{\prime}(0)=-17$.
11. Check whether the solutions $x^{2}$ and $x^{2} \ln x$ are linearly independent.
12. Find the Laplace transform of $a+b t+c t^{2}$.
13. Solve $y^{\prime \prime}+25 y=0$.
14. Find the inverse Laplace transform of $\frac{12}{(s-3)^{4}}$.
15. Write the standard form of Euler Cauchy equation. Give an example.
16. Solve $2 x$ tany $d x+\sec ^{2} y d y=0$.

PART - C
(Essay)
Answer any four questions from this Part. Each question carries 4 marks. ( $4 \times 4=16$ )
17. Find the general solution of $y^{\prime}-y=e^{2 x}$.
18. Solve $y^{\prime \prime}+2 y^{\prime}+y=x^{2}$.
19. Let $f(t)=t$ sinwt, find the Laplace transform of $f(t)$.
20. Check for exactness and solve the initial value problem, $y e^{x} d x+\left(2 y+e^{x}\right) d y=0$, $y(0)=-1$.
21. Solve $y^{\prime}=(y+4 x)^{2}$.
22. Solve $\left(\cot y+x^{2}\right) d x=x \csc ^{2} y d y$.
23. Solve $y^{\prime \prime}+y=\sec x$.

# PART - D <br> (Long Essay) 

Answer any two questions from this Part. Each question carries 6 marks. ( $2 \times 6=12$ )
24. Using Laplace transforms, solve the integral equation, $\mathrm{y}(\mathrm{t})=1-\int_{0}^{\mathrm{t}}(\mathrm{t}-\tau) \mathrm{y}(\tau) \mathrm{d} \tau$.
25. Solve $4 x^{2} D^{2}+24 x D+25 y=0, y(1)=2, y^{\prime}(1)=-6$.
26. Solve $y^{\prime \prime}+2 y^{\prime}+5 y=1.25 e^{0.5 x}+40 \cos 4 x-55 \sin 4 x, y(0)=0.2, y^{\prime}(0)=60.1$.
27. Find an integrating factor and solve the initial value problem

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2 \sin \left(y^{2}\right) d x+x y \cos \left(y^{2}\right) d y=0, y(2)=\sqrt{\frac{\pi}{2}} .
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