Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (C.B.C.S.S.-O.B.E.-Regular/Supplementary/ Improvement) Examination, November 2023
(2019 – 2021 Admissions)
CORE COURSE IN MATHEMATICS
5B06 MAT : Real Analysis – I

Time : 3 Hours   Max. Marks : 48

PART – A

Answer **any 4** questions. They carry **1** mark **each**.   **(4×1=4)**

1. State Triangle Inequality.

2. Find $\lim\left(1+\dfrac{1}{2n}\right)^{n}$.

3. Define m-tail of a sequence.

4. Define continuity of a function at a point.

5. Define Rearrangement of the series.

PART – B

Answer **any 8** questions from among questions **6** to **16**. These questions carry **2** marks **each**.   **(8×2=16)**

6. Determine the set A of $x \in R$ such that $|2x + 3| < 8$.

7. If $a \in R$ and $a \neq 0$ then show that $a^2 > 0$.

8. Discuss the convergence of $\lim\left(\dfrac{n}{2^n}\right)$.

9. Find the limit of the sequence whose terms are given by $x_1 = 8$, $x_{n+1} = \dfrac{x_n}{2} + 2$ for $n \in N$.

10. State Monotone Convergence Theorem.

11. Define subsequence of a sequence with an example.

12. State Alternating Series test.

13. Define convergent Series.

14. If Σa_n with $a_n > 0$ is convergent, then is $\sum \sqrt{a_n}$ always convergent. Justify.

15. Show that $f(x) = \dfrac{1}{x}$ defined on $A = (0, \infty)$ is unbounded on A.

16. State Boundedness Theorem.

PART – C

Answer **any 4** questions from among questions **17** to **23**. These questions carry **4** marks **each**. **(4×4=16)**

17. Show that cosine function is continuous on R.

18. Discuss the convergence of $\sum_{n=0}^{\infty} r^n$, $r \in R$, $|r| < 1$.

19. Discuss the convergence of $\sum_{n=1}^{\infty} \dfrac{n}{n^2+1}$.

20. Discuss the convergence of the sequences
    a) $((-1)^n)$ and
    b) (n).

21. Show that Cauchy sequence of real numbers is bounded.

22. State and prove Archimedean property.

23. If a and b are positive real numbers, $a \neq b$ then show that $\sqrt{ab} \leq \dfrac{(a+b)}{2}$.

PART – D

Answer **any 2** questions from among questions **24** to **27**. These questions carry **6** marks **each**. **(2×6=12)**

24. State and prove density theorem of rational numbers in R.

25. State and prove Squeeze theorem for sequences. Hence find $\lim\left(\dfrac{\sin n}{n}\right)$.

26. Discuss the convergence of
    a) $\sum_{n=0}^{\infty} \dfrac{1}{(n+1)(n+2)}$
    b) $\sum_{n=1}^{\infty} \dfrac{(\cos n)}{n^2}$.

27. Discuss the continuity of
    a) Dirichlet's function
    b) Thomae's function.

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