K23U 2825

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (C.B.C.S.S. – Supplementary)
Examination, November 2023
(2017 and 2018 Admissions)
CORE COURSE IN MATHEMATICS
5B05 MAT : Real Analysis

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** the questions, **each** question carries **one** mark.

1. If $S = \left\{\frac{1}{n} - \frac{1}{m} : n, m \in \mathbb{N}\right\}$, find inf S and sup S.
2. Give an example of two divergent sequences X and Y such that their sum X + Y converges.
3. State comparison test.
4. State Bernstein's approximation theorem. **(4×1=4)**

SECTION – B

Answer **any eight** questions, **each** question carries **two** marks.

5. Prove that there does not exist a rational number r such that $r^2 = 2$.
6. Solve the inequality $|2x - 1| \leq x + 1$.
7. State and prove triangle inequality.

8. Use the definition of the limit of a sequence to establish that $\lim \frac{3n+2}{n+1} = 3$.

9. Let X = (x_n) and Y = (y_n) be sequence of real numbers converge to x and y respectively. Prove that the sequence XY converges to xy.

10. Prove that $\sum_{n=1}^{\infty} \frac{1}{n^2 - n + 1}$ is convergent.

11. Prove that every absolutely convergent series is convergent.

12. State and prove Dirichlet's test.

13. State and prove Bolzano's intermediate value theorem.

14. Define Lipschitz function. If $f : A \to \mathbb{R}$ is a Lipschitz function, prove that f is uniformly continuous on A. **(8×2=16)**

SECTION – C

Answer **any four** questions, **each** question carries **four** marks.

15. State and prove Archimedean property.

16. State and prove density theorem.

17. State and prove Cauchy convergence criterion.

18. State and prove limit comparison test.

19. Let (z_n) be a decreasing sequence of strictly positive numbers with lim(z_n) = 0. Prove that the alternating series $\sum (-1)^{n+1} z_n$ is convergent.

20. Let $I \subseteq \mathbb{R}$ be an interval and let if $f : I \to \mathbb{R}$ be monotone on I. Prove that the set of points $D \subseteq I$ at which f is discontinuous is a countable set. **(4×4=16)**

SECTION – D

Answer **any two** questions, **each** question carries **six** marks.

21. a) State and prove nested interval property.

    b) Prove that the set $\mathbb{R}$ of real numbers is not countable.

22. a) State monotone convergence theorem.

    b) Let $s_1 > 0$ be arbitrary and define $s_{n+1} = \frac{1}{2}\left(s_n + \frac{a}{s_n}\right)$ for $n \in \mathbb{N}$. Prove that (s_n) converges to $\sqrt{a}$.

23. Discuss the convergence of the following series :

    a) $\sum_{n=1}^{\infty} \frac{1}{n!}$,

    b) $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$.

24. State and prove location of roots theorem. **(2×6=12)**