



K23U 2825

Reg. No. :

Name :

V Semester B.Sc. Degree (C.B.C.S.S. – Supplementary)
Examination, November 2023
(2017 and 2018 Admissions)
CORE COURSE IN MATHEMATICS
5B05 MAT : Real Analysis

Time : 3 Hours

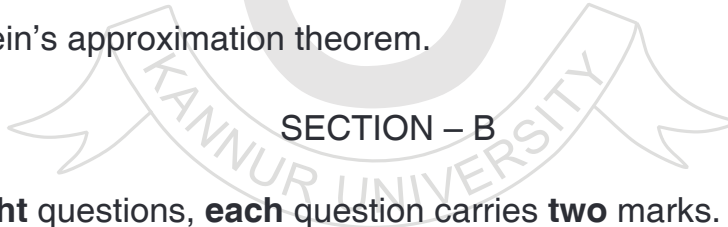
Max. Marks : 48



SECTION – A

Answer **all** the questions, **each** question carries **one** mark.

1. If $S = \left\{ \frac{1}{n} - \frac{1}{m} : n, m \in \mathbb{N} \right\}$, find $\inf S$ and $\sup S$.
2. Give an example of two divergent sequences X and Y such that their sum $X + Y$ converges.
3. State comparison test.
4. State Bernstein's approximation theorem. (4×1=4)



SECTION – B

Answer **any eight** questions, **each** question carries **two** marks.

5. Prove that there does not exist a rational number r such that $r^2 = 2$.
6. Solve the inequality $|2x - 1| \leq x + 1$.
7. State and prove triangle inequality.

P.T.O.



8. Use the definition of the limit of a sequence to establish that $\lim \frac{3n+2}{n+1} = 3$.
9. Let $X = (x_n)$ and $Y = (y_n)$ be sequence of real numbers converge to x and y respectively. Prove that the sequence XY converges to xy .
10. Prove that $\sum_{n=1}^{\infty} \frac{1}{n^2 - n + 1}$ is convergent.
11. Prove that every absolutely convergent series is convergent.
12. State and prove Dirichlet's test.
13. State and prove Bolzano's intermediate value theorem.
14. Define Lipschitz function. If $f : A \rightarrow \mathbb{R}$ is a Lipschitz function, prove that f is uniformly continuous on A . (8×2=16)

SECTION – C

Answer **any four** questions, **each** question carries **four** marks.

15. State and prove Archimedean property.
16. State and prove density theorem.
17. State and prove Cauchy convergence criterion.
18. State and prove limit comparison test.
19. Let (z_n) be a decreasing sequence of strictly positive numbers with $\lim(z_n) = 0$. Prove that the alternating series $\sum (-1)^{n+1} z_n$ is convergent.
20. Let $I \subseteq \mathbb{R}$ be an interval and let if $f : I \rightarrow \mathbb{R}$ be monotone on I . Prove that the set of points $D \subseteq I$ at which f is discontinuous is a countable set. (4×4=16)



SECTION – D

Answer **any two** questions, **each** question carries **six** marks.

21. a) State and prove nested interval property.

b) Prove that the set \mathbb{R} of real numbers is not countable.

22. a) State monotone convergence theorem.

b) Let $s_1 > 0$ be arbitrary and define $s_{n+1} = \frac{1}{2} \left(s_n + \frac{a}{s_n} \right)$ for $n \in \mathbb{N}$. Prove that (s_n) converges to \sqrt{a} .

23. Discuss the convergence of the following series :

a) $\sum_{n=1}^{\infty} \frac{1}{n!}$,

b) $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$.

24. State and prove location of roots theorem.

(2×6=12)

